HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS

2005

YEAR 12

ASSESSMENT TASK 2

HALF YEARLY EXAMINATION

MATHEMATICS

EXAMINERS ~ S. GEE, P. BIZCO, H. CAVANAGH, S. FAULDS, R. YEN

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working Time – 2 hours.
- Attempt **all** questions.
- Questions are of equal value.
- **All** necessary working should be shown in every question.
- This paper contains ten (10) questions.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators and MathAids may be used.
- **Each question is to be started in a new answer booklet**.
- This examination paper must **NOT** be removed from the examination room

STUDENT NAME: ____________________________

TEACHER: ____________________________

Start each question on a separate writing booklet.

QUESTION ONE **8 marks** Start a SEPARATE booklet **Marks**

(a) Find the value of $\sqrt{\dfrac{19}{4\pi}}$ correct to 2 decimal places **1**

(b) Simplify $\dfrac{m+3}{2} - \dfrac{m+2}{3}$ **2**

(c) Factorise $3k^2 - 8k - 3$ **2**

(d) A 2.5% increase in the annual Council rates increased the charge by \$28. What was the *original* charge? **1**

(e) Factorise $a^3 - 27$ **2**

QUESTION TWO. **8 marks** Start a NEW booklet.

(a) Solve the following equation for $0° \le \theta \le 360°$:

$$\sqrt{3}\tan\theta = -1$$ **3**

(b) Prove the identity:

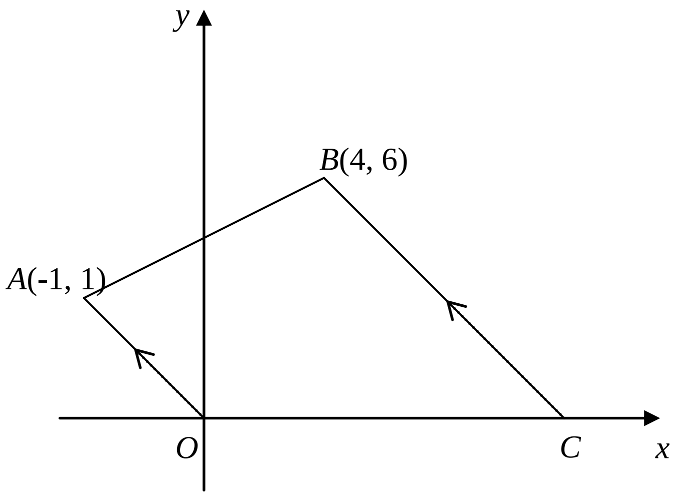
$$5 - 5\sin^2\theta \equiv 5\cos^2\theta$$ **2**

(c) A ship is travelling due west at 20 knots.
From a point A, a lighthouse is sighted on a bearing of 300°.
Two hours later, at point B, the lighthouse can be seen on a bearing of 345°.

(i) Draw a neat diagram which illustrates the information given above. **1**

(ii) How far is the point B from the lighthouse? **2**
Give your answer to the nearest nautical mile.

QUESTION THREE. **8 marks** Start a NEW booklet. **Marks**



In the diagram, *OABC* is a trapezium with *OA* || *CB*. The coordinates of *O*, *A* and *B* are (0, 0), (-1, 1) and (4, 6) respectively.

(a) Calculate the length of *OA*. **1**

(b) Write down the gradient of the line *OA*. **1**

(c) Find the equation of the line *BC*. **1**

(d) Find the coordinates of *C*. **1**

(e) Show that the perpendicular distance from *O* to the line *BC* is $5\sqrt{2}$. **2**

(f) Hence, or otherwise, calculate the area of the trapezium *OABC*. **2**

QUESTION FOUR. **8 marks** Start a NEW booklet.

Consider the curve given by $y = \frac{1}{4}x^4 - x^3$.

(a) Find any turning points and determine their nature. **3**

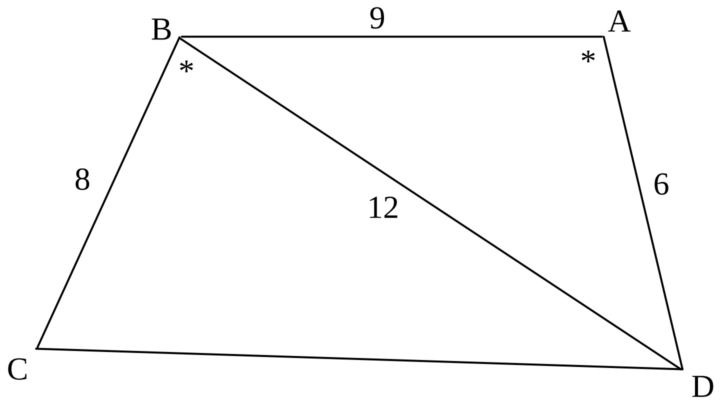
(b) Find any points of inflexion. **2**

(c) Sketch the curve for $-1 \leq x \leq 4$. **1**

(d) For what values of *x* is the curve concave down? **2**

QUESTION FIVE **8 marks** Start a SEPARATE booklet **Marks**

(a)



$\angle DAB = \angle CBD$

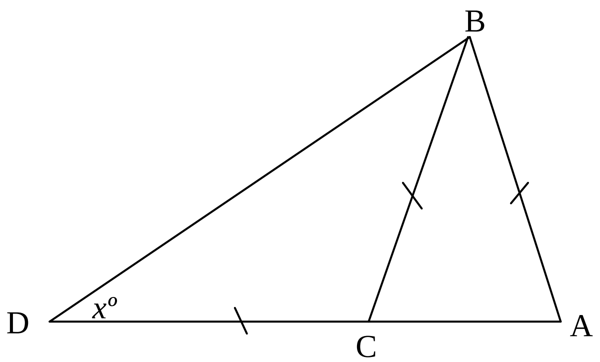
(NOT TO SCALE)

(i) Prove triangles ABD and BDC are similar. **2**

(ii) Find the length of CD. **1**

(iii) Prove that AB and CD are parallel. **1**

(b)



AB = BC = CD; $\angle BDC = x^o$

(i) Prove that $\angle CAB = 2x^o$ **2**

(ii) If $\angle ABD = 120^o$, find the value of x. **2**

QUESTION SIX **8 marks** Start a SEPARATE booklet **Marks**

(a) Three consecutive terms of a sequence are $2x + 5$, T_2 and $8x + 19$.
Find T_2 in terms of x if the sequence is to be arithmetic. **2**

(b) For the sequence 3, $\frac{11}{2}$, 8, … find:

(i) the 37th term **2**

(ii) the sum of 37 terms. **2**

(c) Does the sequence $\frac{3}{4}, 1, \frac{4}{3}, \ldots$ have a limiting sum?
Explain your answer, stating S_∞ if it exists. **2**

QUESTION SEVEN **8 marks** Start a SEPARATE booklet

(a) Show the equation of a parabola is $x^2 - 2x - 2y - 13 = 0$ is also given by
$(x-1)^2 = 2(y+7)$. **1**

Find:

(i) the coordinates of its vertex **1**

(ii) the focal length **1**

(iii) the equation of its directrix **1**

(b) $A(1, 0)$ and $B(4, 0)$ are points on the number plane.
The point $P(x, y)$ moves such that the length of PB is twice the length of PA.

(i) Write a formula for the length of PB. **1**

(ii) Prove that the locus of P is a circle and determine its centre and radius. **3**

QUESTION EIGHT **8 marks** Start a SEPARATE booklet

(a) If $2x^2 - 7x + 4 = a(x+2)^2 + b(x+2) + c$ for all values of x, find a, b and c. **2**

(b) Find all values of k for which the quadratic equation $x^2 + (k-3)x + k = 0$ has real roots **3**

(c) Find all real numbers x which satisfy the equation $x^4 = 4(x^2 + 8)$ **3**

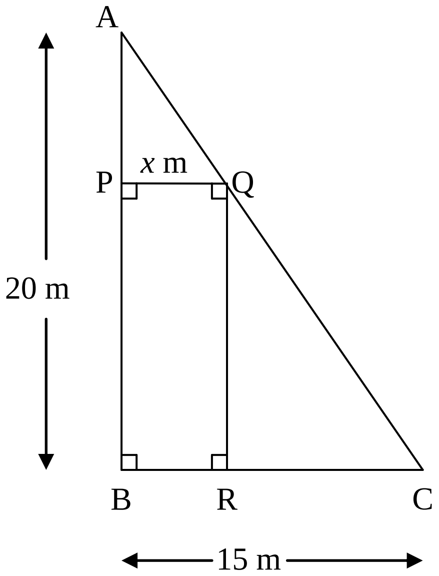
QUESTION NINE **8 marks** Start a SEPARATE booklet **Marks**

(a) For the function $y = x + \frac{900}{x}$

(i) Find $\frac{dy}{dx}$ **1**

(ii) Show that y has a relative minimum value of 60. **2**

(b) In the triangle ABC, AB = 20m, BC = 15m and angle ABC = $90°$. BPQR is a rectangle inscribed in ABC, as shown, with PQ = x metres.



(i) Prove that ΔAPQ ||| ΔABC **1**

(ii) Find the length of AP in terms of x and hence show that the area of the rectangle BPQR is given by $x(20 - \frac{4x}{3})$ m^2 **2**

(iii) Hence find the maximum possible area of the rectangle BPQR **2**

QUESTION TEN **8 marks** Start a SEPARATE booklet

(a) Find the primitive of $x^2 + 2x - 3$. **1**

(b) Expand and simplify $(x^2 + 2)^2$. **2**
Hence find the primitive of $(x^2 + 2)^2$.

(c) The curve $y=f(x)$ has a gradient function **3**

$$\frac{dy}{dx} = 3x^2 - 2x + 1.$$

If the curve passes through the point Q(2,3), find its equation.

(d) Find the domain for which $y = \frac{1}{x^2 + 1}$ is a decreasing function. **2**

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Question No. 1 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

P3 *performs routine arithmetic and algebraic manipulation involving surds and simple rational expressions.*

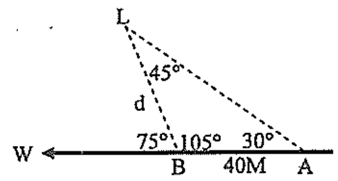
Outcome	Solutions	Marking Guidelines
P3	(a) $\sqrt{\frac{19}{4\pi}} = 1.23$ (2 d.p's)	<u>1 mark</u> – correct answer
	(b) $\frac{m+3}{2} - \frac{m+2}{3} = \frac{3(m+3)-2(m+2)}{6}$ $= \frac{3m+9-2m-4}{6}$ $= \frac{m+5}{6}$	<u>2 marks</u> – fully correct <u>1 mark</u> – correct working but for 1 error.
	(c) $3k^2 - 8k - 3 = (3k+1)(k-3)$	<u>2 marks</u> – both factors correct
	(d) 2.5% → \$28 1% → \$11.20 ∴ 100% → \$1120 (original charge)	<u>1 mark</u> – correct solution
	(e) $a^3 - 27 = (a-3)(a^2+3a+9)$	<u>2 marks</u> – both factors correct. <u>1 mark</u> – 1 factor correct, but no marks awarded for $(a-3)^3$

Year 12 Mathematics Half Yearly Examination 2005

Question No. 2 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

Outcome	Solutions	Marking Guidelines
H5	(a) $\sqrt{3}\tan\theta = -1$ $\tan\theta = -\frac{1}{\sqrt{3}}$ $\therefore \theta = 150°, 330°$ (*Basic angle is 30°, tan is negative in 2nd and third quadrants*)	**3 marks** Correct answer, showing basic angle and correct quadrants. **2 mark** Only one correct angle shown, OR Incorrect basic angle but answers given in correct quadrants **1 mark** Some reference made to the correct basic angle of 30° by rearranging equation.
H5	(b) $5 - 5\sin^2\theta \equiv 5\cos^2\theta$ $L.H.S = 5 - 5\sin^2\theta$ $= 5(1-\sin^2\theta)$ $= 5\cos^2\theta$ (*since* $\sin^2\theta + \cos^2\theta = 1$) $= R.H.S$	**2 marks** Correct use of identities with working set out in logical, ordered way as required for a proof. **1 mark** Correct use of identities but not presented as a proof, OR Demonstrates knowledge of at least one relevant identity in context.
H5	(c) (i) 	**1 mark** Diagram in correct orientation shows correct angles in correct positions (∠ABL or reflex ∠ABL <u>and</u> ∠LAB or reflex ∠LAB <u>and</u> AB=40M. **0 marks** any of the above incorrect or missing
H5	(ii) $\frac{d}{\sin 30°} = \frac{40}{\sin 45°}$ $d = \frac{40\sin 30°}{\sin 45°}$ $= \frac{40\times\frac{1}{2}}{\frac{\sqrt{2}}{2}}$ $= \frac{40}{\sqrt{2}}$ $\approx 28M$ (*to nearest nautical mile*)	**2 marks** Correct use of sine rule to obtain correct answer (rounding disregarded). Note: correct answer may be obtained using information from incorrect answer in (i) above. **1 mark** Sine rule used correctly with arithmetic error causing incorrect answer, OR Sine rule used correctly but with wrong angles giving incorrect answer or wrong distance found.

Year 12 Mathematics Half-Yearly Examination 2005
Question No. 3
Solutions and Marking Guidelines by R. Yen

Outcomes Addressed in this Question

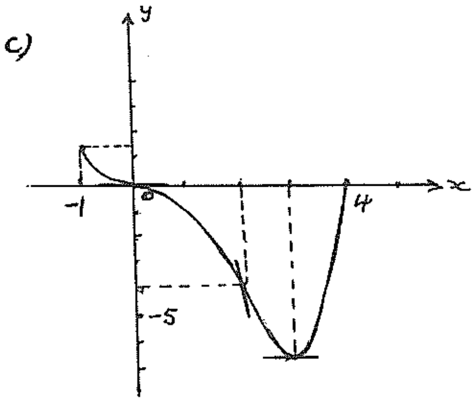
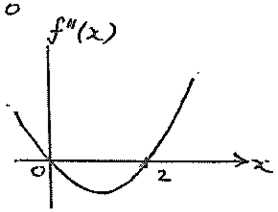
P4 chooses and applies appropriate arithmetic, algebraic and graphical techniques

Outcome	Solutions	Marking Guidelines
P4	(a) $OA = \sqrt{(-1-0)^2+(1-0)^2}$ $= \sqrt{1+1} = \sqrt{2}$ units	✓ ①
	(b) $M_{OA} = \frac{1-0}{-1-0} = \frac{1}{-1} = -1$	✓ ①
	(c) $M_{BC} = M_{OA} = -1$ (OA ∥ CB) $y-6 = -1(x-4)$ $= -x+4$ $y = -x+10$ (or $x+y-10=0$)	✓ ①
	(d) When $y=0$: $0 = -x+10$ $x = 10$ ∴ C is (10, 0)	✓ ①
	(e) BC has general equation $x+y-10=0$ $d = \frac{\lvert 0+0-10\rvert}{\sqrt{1^2+1^2}}$ [sub O(0,0)] $= \frac{10}{\sqrt{2}}$ $= \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ ← $= \frac{10\sqrt{2}}{2}$ $= 5\sqrt{2}$ units	✓ ① correct substitution into perpendicular distance formula must show this step because the answer $5\sqrt{2}$ is given in the question ✓ ①
	(f) $BC = \sqrt{(6-0)^2+(4-10)^2}$ $= \sqrt{72}$ $= 6\sqrt{2}$ units Area $= \frac{1}{2}(OA+BC)d$ $= \frac{1}{2}(\sqrt{2}+6\sqrt{2})5\sqrt{2}$ $= \frac{1}{2}(7\sqrt{2})5\sqrt{2}$. $= \frac{1}{2} \times 35 \times 2$ $= 35$ units^2	✓ ① length of BC as a surd ✓ ① correct working to find area

Year 12 Mathematics Half Yearly Examination 2005
Question No. 4
Solutions and Marking Guidelines

Outcomes Addressed in this Question

P6 P7 P8 H2 H6 H5

Outcome	Solutions	Marking Guidelines
P7 H5 H6	a) $y = \frac{1}{4}x^4 - x^3$ $y' = x^3 - 3x^2 = 0$ a T.Pts. $x^2(x-3) = 0$ $x = 0\}$ $3\}$; $y = 0\}$ $-6.75\}$ $y'' = 3x^2 - 6x = 0$ at (0, 0) ∴ Possible horizontal Inflexion at (0,0) $x = 0^-$ $y'' > 0$, $x = 0^+$ $y'' < 0$ ∴ H.P.o.I. $x = 3^-$ $y'' > 0$, $x = 3^+$ $y'' > 0$ ∴ Rel. Min. The only turning pt is a Rel. Min. at (3, −6.75)	Demonstrate ability to differentiate and set derivative to zero – (1 mark). Demonstrate ability to locate possible t. points (1 mark) Identify the t. pt. as a relative min (1 mark)
H5 H6	b) $y'' = 3x^2 - 6x = 0$ at P.O.I. $3x(x-2) = 0$ $x = 0\}$ $2\}$; $y = 0\}$ $-4\}$ H.P.O.I. already found at (0, 0) $x = 2^-$ $y'' < 0$ $x = 2^+$ $y'' > 0$ Change in concavity ∴ P.O.I at (2, −4)	Demonstrate knowledge for locating Pts. of Inflex. (1 mark) Successfully locate P.O.I. at (0, 0) and (2, −4) — (1 mark)
H6 P6	c) 	Demonstrate ability to represent data derived in (a) and (b) on a cartesian plane (1 mark)
P8	d) f(x) concave down when $y'' < 0$ $3x^2 - 6x < 0$ $x^2 - 2x < 0$ $x(x-2) < 0$  $f''(x) < 0$ when $0 < x < 2$ Thus f(x) is concave down f $0 < x < 2$	Knowledge of condition for f(x) to be concave down (1 mark). Demonstrate ability to solve the quadratic inequation (1 mark) OR Demonstrate ability to correctly interpret the graph (2 marks)

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Question No. 5 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

H2 Constructs arguments to prove and justify results

Outcome	Solutions	Marking Guidelines
H2 (a) (i)	$\angle DAB = \angle CBD$ (given) $\frac{AD}{AB} = \frac{6}{9} = \frac{2}{3}$ $\frac{BC}{BD} = \frac{8}{12} = \frac{2}{3}$ $\Delta ADB \parallel\!\mid \Delta BDC$ (one pair of included angles equal and the sides about them in are in the same ratio)	1 mark for proportionality statement 1 mark for statement with the correct reason
H2 (ii)	$\frac{DC}{BD} = \frac{BD}{AD}$ (sides are in the same proportion) $\frac{DC}{12} = \frac{12}{9}$ $DC = 16$ cm	1 mark for correct answer
H2 (iii)	$\angle ABD = \angle BDC$ (corresponding angles in similar triangles) $AB \parallel CD$ (alternate angles are equal)	1 mark for correct solution (with appropriate reasons stated)
H2 (b) (i)	$\angle DBC = x$ (angles opposite equal sides) $\therefore \angle BCA = 2x$ (exterior angle of ΔBCD) $\therefore \angle BAC = 2x$ (angles opposite equal sides)	1 mark for determining the size of $\angle BCA$ 1 mark for determining the size of $\angle BAC$
H2 (ii)	Since $\angle ABD = 120^0$ $x + 2x + 120 = 180$ (Angle sum of ΔABD) $3x = 60$ $x = 20$	1 mark for correct statement with correct reason 1 mark for correct answer

Year 12 Mathematics Half Yearly Examination 2005

Question No. 6 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

Outcome	Solutions	Marking Guidelines
H5	(a) For an arithmetic sequence: $T_3 - T_2 = T_2 - T_1$ $\therefore 8x + 19 - T_2 = T_2 - (2x - 5)$ $= T_2 - 2x + 5$ $10x + 24 = 2T_2$ *ie.* $T_2 = 5x + 12$	**2 marks** Correct expression for T_2 given with appropriate working. **1 mark** In correct expression for T_2 found due to algebraic error in working, OR Uses test for arithmetic sequence in attempting to find an expression for T_2.
H5	(b) (i) $T_3 - T_2 = 8 - \frac{11}{2}$ $= \frac{5}{2}$ $T_2 - T_1 = \frac{11}{2} - 3$ $= \frac{5}{2}$ $\therefore$ *Sequence is arithmetic with* $d = \frac{5}{2}$ $T_n = a + (n-1)d$ $T_{37} = 3 + 36 \times \frac{5}{2}$ $= 93$	**2 marks** Correctly identifies sequence as arithmetic, finding the value of d, and hence finding a correct value for T_{37}, OR Uses other valid process to find T_{37}. **1 mark** Finds incorrect value of d leading to a consistent value of T_{37}, OR Finds a correct value of d without valid justification. **0 marks** Incorrect value of T_{37} with no valid working.
H5	(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{37} = \frac{37}{2}[2 \times 3 + (37-1) \times \frac{5}{2}]$ $= 18\frac{1}{2} \times 96$ $= 1776$	**2 marks** Correctly states formula for sum of an arithmetic sequence, following on to find a correct answer for S_{37}, OR Uses correct formula with incorrect information from (i) above leading to a consistent answer. **1 mark** Correct formula and information from (i) above used with arithmetic error in calculation.
H5	(c) For a geometric series: $\frac{T_3}{T_2} = \frac{T_2}{T_1} = r$ $\frac{T_2}{T_1} = \frac{\frac{4}{3}}{1}$ $= \frac{4}{3}$ $\frac{T_3}{T_2} = \frac{1}{\frac{3}{4}}$ $= \frac{4}{3}$ $\therefore$ *Sequence is geometric with* $r = \frac{4}{3}$ *Here, there is no limiting sum since* $\lvert r \rvert > 1$.	**2 marks** Correctly finds the value of the common ratio and states that a sum to infinity does not exist with valid reason. **1 mark** Finds the correct value of the common ratio and then finds sum to infinity despite its non-existance. **0 marks** States that sum to infinity does not exist without justification.

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Question No. 7 | Solutions and Marking Guidelines

Outcomes Addressed in this Question

P 3 performs routine arithmetic and algebraic manipulation involving surds, **simple rational expressions** and trigonometric identities
P 4 chooses and applies appropriate arithmetic, **algebraic**, graphical, trigonometric and **geometrical techniques**

Outcome	Solutions	Marking Guidelines
P 4	(a) $x^2-2x-2y-13=0$ $\therefore x^2-2x=2y+13$ $\therefore x^2-2x+1=2y+13+1$ $\therefore (x-1)^2=2(y+7)$	1 mark : correct solution
P 3	(i) (1, -7)	1 mark : correct solution
P 3	(ii) $4a=2$ $\therefore a=\frac{1}{2}$	1 mark : correct solution
P 4	(iii) directrix is $\frac{1}{2}$ a unit below the vertex (1, -7). $\therefore$ directrix is $y=-7\frac{1}{2}$	1 mark : correct solution or correct reasoning
P 4	(b) (i) $PB=\sqrt{(x-4)^2+y^2}$	1 mark : correct use of distance formula
P 4	(ii) PB=2PA $\therefore \sqrt{(x-4)^2+y^2}=2\sqrt{(x-1)^2+y^2}$ $\therefore (x-4)^2+y^2=4((x-1)^2+y^2)$ $\therefore x^2-8x+16+y^2=4x^2-8x+4+4y^2$ $\therefore 3x^2+3y^2=12$ $\therefore x^2+y^2=4$ which is a circle of centre (0, 0) and radius 2 units	3 marks : correctly squaring both sides of correct equation linking PA and PB, plus correctly simplifying to give equation of a circle, plus finding radius and centre of the circle 2 marks : two of above 1 mark : one of above

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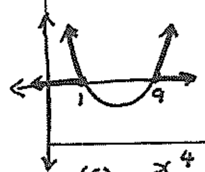
Question No. 8 | Solutions and Marking Guidelines by R. Yen

Outcomes Addressed in this Question

P4 chooses and applies appropriate arithmetic and algebraic techniques

Outcome	Solutions	Marking Guidelines
P4	(a) $2x^2-7x+4=a(x+2)^2+b(x+2)+c$ $=a(x^2+4x+4)+bx+2b+c$ $=ax^2+4ax+4a+bx+2b+c$ $=ax^2+(4a+b)x+4a+2b+c$ $\therefore a=2$ $4a+b=-7$ $4a+2b+c=4$ $4(2)+b=-7$ $4(2)+2(-15)+c=4$ $8+b=-7$ $8-30+c=4$ $b=-15$ $c=26$ $\therefore$ $a=2,\ b=-15,\ c=26$ [ALTERNATIVE SOLUTION BY SUBSTITUTING VALUES :	✓ (1) correct expansion ✓ (1) correct answer (1) correct simultaneous equations]
	(b) Real roots if $\Delta \geq 0$ $\Delta = b^2-4ac \geq 0$ $(k-3)^2-4.1.k \geq 0$ $k^2-6k+9-4k \geq 0$ $k^2-10k+9 \geq 0$ $(k-9)(k-1) \geq 0$ $k \leq 1$ or $k \geq 9$  	✓ (1) correct quadratic inequality $\geq$ ✓ (1) correct factorisation ✓ (1) correct solution
	(c) $x^4=4(x^2+8)$ $=4x^2+32$ $x^4-4x^2-32=0$ $(x^2-8)(x^2+4)=0$ $x^2=8$ or $x^2=-4$ $x=\pm\sqrt{8}$ No solution $=\pm 2\sqrt{2}$ $\therefore x=\pm 2\sqrt{2}$	✓ (1) make equation = 0 ✓ (1) consider 2 cases ✓ (1) correct solution

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Question No. 9 Solutions and Marking Guidelines

Outcomes Addressed in this Question

- P 3 performs routine arithmetic and algebraic manipulation involving surds, **simple rational expressions** and trigonometric identities
- H 2 constructs arguments to prove and justify results
- H 5 applies appropriate techniques from the study of **calculus**, geometry, probability, trigonometry and series to solve problems
- H 6 uses the derivative to determine the features of the graph of a function
- H 9 communicates using **mathematical language, notation,** diagrams and graphs

Outcome	Solutions	Marking Guidelines
H 5	(a) (i) $y = x + \frac{900}{x} = x + 900x^{-1}$ $\frac{dy}{dx} = 1 - 900x^{-2} = 1 - \frac{900}{x^2}$	1 mark : correct answer
H 6, H 9	(ii) Maximum/Minimum when $\frac{dy}{dx} = 0$ $1 - \frac{900}{x^2} = 0$ when $x^2 - 900 = 0$, giving $x = \pm 30$ $\frac{d^2y}{dx^2} = 1800x^{-3} = \frac{1800}{x^3}$ When $x = 30$, $\frac{d^2y}{dx^2} = \frac{1800}{30^3} > 0$ $\therefore$ minimum when $x = 30$ When $x = 30$, $y = 30 + \frac{900}{30} = 60$ $\therefore$ y has a relative minimum value of 60	1 mark : solving $\frac{dy}{dx} = 0$ and finding corresponding y value 1 mark : justify minimum, by finding $\frac{d^2y}{dx^2}$ or using 1st derivative test
H 2	(b) (i) $\angle$A is common $\angle$APQ = $\angle$ABC (both right angles) $\angle$AQP = $\angle$ACB (corresponding, PQ‖BC $\therefore$ ΔAPQ is similar to ΔABC	1 mark : state 2 pairs of equal angles, with reason and justify equiangular
P 3	(ii) as Δ's similar, sides in same ratio $\therefore \frac{AP}{20} = \frac{x}{15}$ $\therefore$ AP $= \frac{4x}{3}$ Area rectangle BPQR = PB $\times$ PQ $= (20 - \frac{4x}{3}) \times x$ $= x(20 - \frac{4x}{3})$	1 mark : correct answer AP 1 mark : justify area using length $\times$ width (provided didn't contradict AP)
H 6, H 9	(iii) A $= x(20 - \frac{4x}{3}) = 20x - \frac{4x^2}{3}$ $A' = 20 - \frac{8x}{3} = 0$ for maximum/minimum which occurs when $x = 7.5$ $A'' = -\frac{8}{3} < 0$ $\therefore$ maximum when $x = 7.5$ Maximum area $= 7.5(20 - \frac{4 \times 7.5}{3}) = 75\,m^2$.	1 mark : correctly find A' and solve 1 mark : test maximum and find corresponding area

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Question No. 10 Solutions and Marking Guidelines

Outcomes Addressed in this Question

P4 P5 P8 H5 H6

Outcome	Solutions	Marking Guidelines
H5	a) $f(x) = x^2 + 2x - 3$ Primitave is $x^3/3 + x^2 - 3x + c$	Correct answer (1 mark)
P4	b) $(x^2+2)^2 = (x^2)^2 + 2(x^2)(2) + 2^2$ $= x^4 + 4x^2 + 4$ Primitive is $x^5/5 + 4/3\,x^3 + 4x + c$	Demonstrate ability to square the binomial (1 mark) Correct answer (1 mark)
H6	c) $\frac{dy}{dx} = 3x^2 - 2x + 1$ $y = x^3 - x^2 + x + c$ Curve passes through (2,3) Substitute $x=2$, $y=3$ and solve to find c. $3 = 8 - 4 + 2 + c$ $c = 3 - 8 + 4 - 2 = -3$ $\therefore$ Equation of the curve is $y = x^3 - x^2 + x - 3$.	Correct primitive (1 mark) Knowledge of condition for curve to pass through (2,3) - (1 mark) Correct evaluation of c (1 mark)
P5 P8	d) $y = \frac{1}{x^2+1}$ $y' = \frac{-2x}{(x^2+1)^2}$ Decreasing function when $y' < 0$ $(x^2+1)^2 > 0$ for all x $\therefore \frac{-2x}{(x^2+1)^2} < 0$ for all $x > 0$ i.e. $y = \frac{1}{x^2+1}$ is a decreasing function when $x > 0$	Correct derivative (1 mark) Correct domain (1 mark)